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22nd Primary Mathematics World Contest

Individual Contest 2019 – Shortlisted Questions

Individual Contest

Question 1

The positive integers starting from 1, are written consecutively as "123456789101112...". What is the 2019th digit?

Question 2

A father has *n* children. In his will, he distributes all of his money to his children in the following manner:

(1) to his first child he gives \$1000 and $\frac{1}{10}$ of his remaining money;

(2) then to his second child he gives \$2000 and $\frac{1}{10}$ of his remaining money;

(3) then to his third child he gives \$3000 and $\frac{1}{10}$ of his remaining money;

……and so on to the rest of his children until no money remains (The last child receives $n \times \$1000$). Find *n* if each child has received the same amount of money.

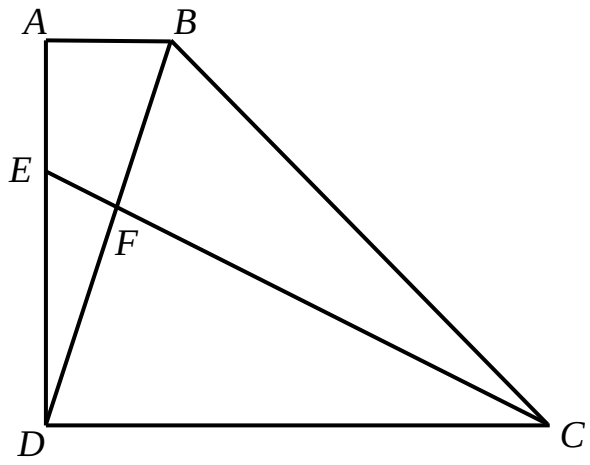
Question 3

A shopkeeper buys two products, A and B, at a total cost of $200. He decides to sell product A with a 30% profit and product B with a 20% profit. Later, he has a sale and discounts the selling price of each product by 10%. Finally, after selling both products, he has made a total profit of $27.70. How much did the shopkeeper originally pay for product A?

Question 4

In the figure shown below, *ABCD* is a right-angled trapezoid.

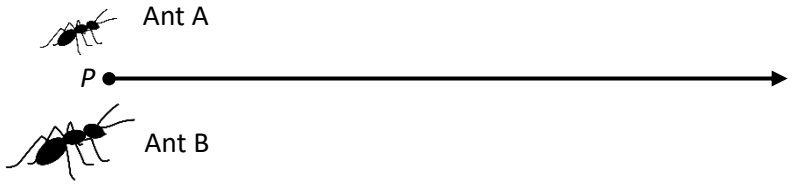
If $AB = AE = 4$ cm, $DE = 8$ cm and $CD = 16$ cm, what is the area, in cm^2, of triangle *BCF*?



Question 5

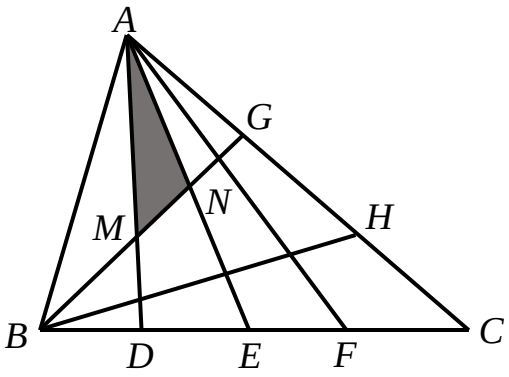
Starting at point *P*, ants A and B are crawling in a straight line. Ant A crawls at 3 cm per second and ant B crawls at 5 cm per second. Ant A starts crawling first. After it has crawled 2 cm, ant B starts to chase it. When ant B catches up to ant A it turns around and crawls back to point *P*. It then turns back and starts to chase ant A again.

What is the total distance, in cm, that ant B has crawled when it catches up to ant A for the third time?



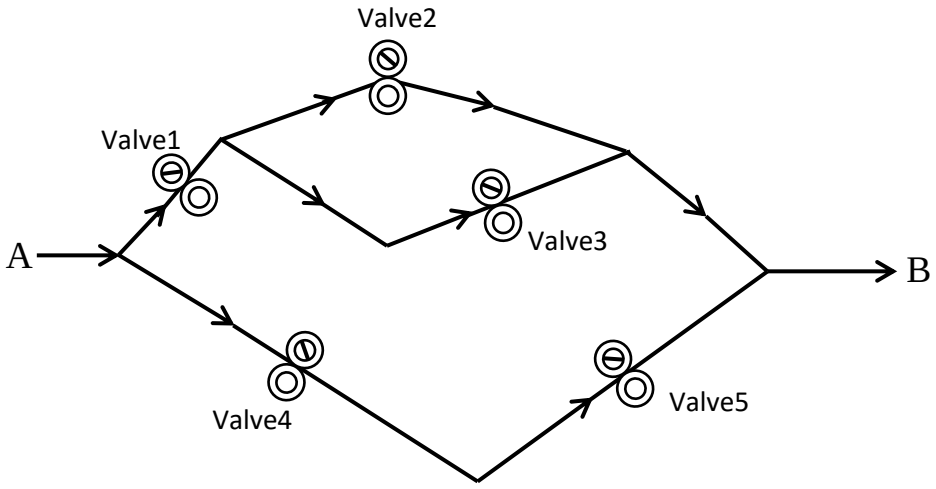
Question 6

In triangle *ABC* below, $BD = DE = EF = FC$ and $AG = GH = HC$. If the area of triangle *AMN* is 3.5 cm^2, then what is the area, in cm^2, of triangle *ABC*?



Question 7

Water flows through a network of pipes in the direction shown in the figure from A to B. Five valves on the network as shown can be open (◎) to allow water to flow through or closed (⊘) to stop the flow of water. There are $2^5 = 32$ different ways of setting the valves. How many of these 32 ways will allow water to flow from A to B ?

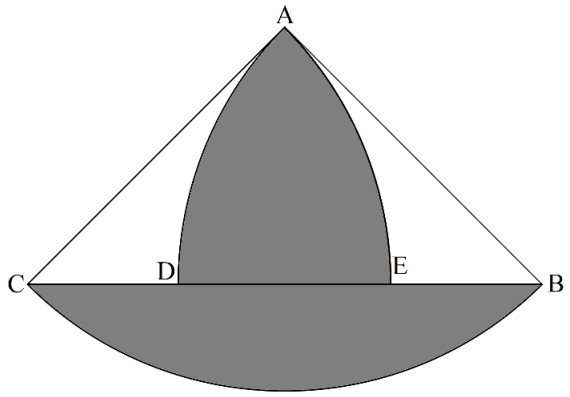


Question 8

In a school, the sum of a number of students' Mathematics exam scores is 8250. The top three students scored 88, 85 and 80, respectively. All of the scores are positive integers. If there are no more than 3 students having the same score, and the lowest score is 30, then what is the minimum number of students to score 60 or more?

Question 9

In the figure below, *ABC* is a right-angled isosceles triangle where $\angle CAB = 90°$ and *AB* = 28 cm. Using points *A*, *B* and *C* as the centres, three arcs of radius 28 cm are drawn as shown.
What is the area, in cm^2, of the shaded region?
(Use $\pi = \frac{22}{7}$)



Question 10

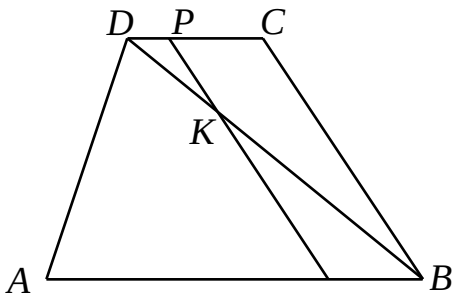
If $\overline{AB}$ and $\overline{CB}$ are two-digit positive integers such that

$\overline{AB} \times \overline{CB} = \overline{BCBB}$, then what is the maximum value of $\overline{AB} + \overline{CB}$?

(*A*, *B* and *C* each represents different digits.)

Question 11

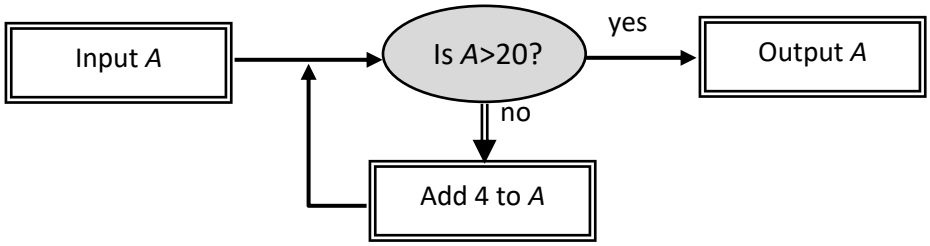
In trapezoid *ABCD* below, point *K* lies on the diagonal *BD* and *DK* : *KB* = 1 : 3. A line parallel to *BC* goes through point *K* and meets *DC* at point *P*. If *AB* = 4 cm and *CD* = 1 cm, then what is the ratio of the areas of *BCPK* to *ABCD*?



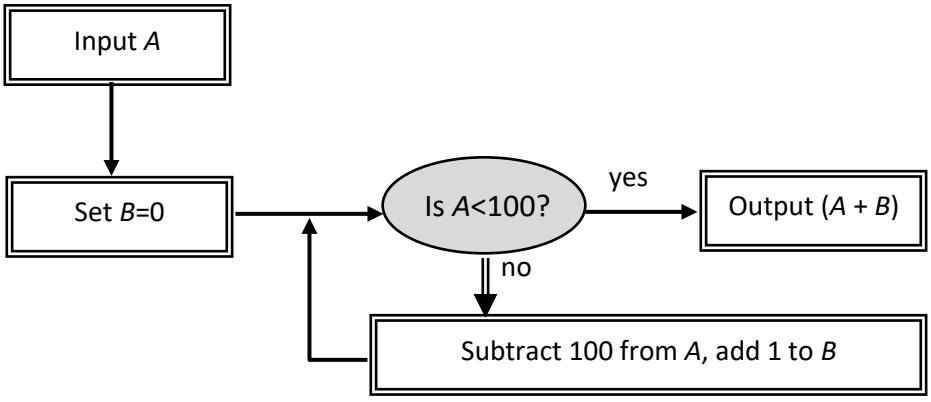
Question 12

Flow diagrams provide a visual way of showing a process or algorithm: a box is used for an action, an oval (shaded) for making a decision and arrows indicate the flow of control.

For example, in the flow diagram below, if *A* was input as 19, then the output would be 23, whilst if it was input as 21, then it would be unchanged and the output would be 21.



If 2019 is input to the flow diagram below, then find the value of the output.



Question 13

If the pattern in the triangular array below is continued, what number would be directly above 165?

1

2 3 4

5 6 7 8 9

10 11 12 13 14 15 16

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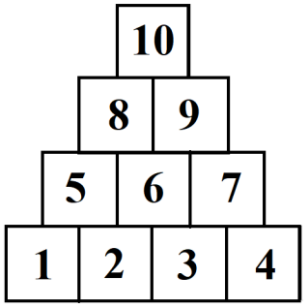
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and so on…

Question 14

In the figure below, starting from box 1, you are only allowed to move to an adjacent box of a higher number. For example, from box 1 you have one way to move to box 2, but two ways to move to box 5 ($1 \rightarrow 5$ or $1 \rightarrow 2 \rightarrow 5$). In how many different ways can you move from box 1 to box 10?



Question 15

Four students, *A*, *B*, *C* and *D* sit around a table. There is a jar of marbles and an empty box on the table. First, student *A* takes a marble from the jar and puts it into the box, then takes a quarter of the remaining marbles. Next, student *B* takes a marble from the remaining jar and puts it into the box, then takes a quarter of the remaining marbles. Then students *C* and *D* also do the same respectively. Finally, all of the remaining marbles are divided equally among the four students. What is the minimum number of marbles that were on the table to start with?